# Technical Comments\_

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# **Comment on "Flutter Prediction** from Flight Flutter Test Data"

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#### Introduction

N a previous paper entitled "Flutter Predictions from Flight Flutter Test Data," the authors applied a number of different flutter prediction methods to data from two simulated aeroelastic aircraft models and compared the resulting flutter predictions. The two simulated models were a simple three-degree-of-freedom Hancock wing model and the Sim-2 model of a generic four-engined civil transport. One of the methods examined in the paper was the Nissim and Gilyard method (NGM).<sup>2,3</sup> Because of difficulties encountered with the Sim-2 model, the authors failed to apply the NGM successfully to it, and only results for the Hancock model were presented in the paper. With the aid of Eli Nissim, the authors have now succeeded in applying the method to the Sim-2 model and to obtain quality flutter predictions from it. In this short Comment, the initial problems encountered will be described and then the solutions will be outlined. Finally, flutter predictions for the method will be presented and compared to the flutter predictions obtained from the other methods by Dimitriadis and Cooper.1

## Numerical Problems with the Sim-2 Model and Solutions

The Sim-2 model is described in detail in Ref. 1. There are two sets of matrices, one for symmetric modes (21 flexible and 2 rigid body) and one for antisymmetric modes (21 flexible and 3 rigid body). The symmetric and antisymmetric behaviors are uncoupled and can be solved separately. In physical space, the aircraft can be excited at 17 excitations positions.

Dimitriadis and Cooper<sup>1</sup> state that the low-frequency rigid-body modes render the equations of motion quite "stiff." In fact, the Sim-2 model exhibited large rigid-body responses to forced dynamic excitations, masking in many cases the elastic responses. This in turn creates problems with the application of the NGM because it renders the data matrix in the identification process badly scaled, even in the absence of noise. The rigid-body modes were not a problem with any of the other methods for the following reasons:

- 1) The flutter margin and damping fit methods are based on the calculation of the system eigenvalues from the frequency response functions. This calculation can take place at any frequency range of interest, thus excluding the effects of the rigid-body
- 2) The envelope function can be calculated from time-domain acceleration, velocity, or displacement signals. The rigid-body modes add approximately linear components to the displacement signals. These are not present in the acceleration signals after being differentiated twice. Therefore the envelope function can be calculated from acceleration data, thus, avoiding any problems associated with the rigid-body modes.
- 3) The auto-regressive moving average (ARMA)-based method can also be applied to either acceleration, velocity, or displacement responses. As with the envelope function, if it is applied to acceleration signals, then the rigid-body issues are avoided.

The Nissim and Gilyard technique requires acceleration, velocity, and displacement measurements to work; therefore, it is the only method affected by the rigid-body issues. After consultation with Nissim, these issues were resolved by the following two means.

- 1) The rigid-body modes were removed from the equations of motion. This allowed for a perfect identification of the flexible modes in the absence of noise.
- 2) Because the responses associated with the higher-frequency modes are much smaller than those associated with the lowfrequency modes, these latter modes are easily corrupted by the introduction of noise. This was overcome following Nissim and Gilyard's suggestion<sup>2</sup> to weigh the data matrix in the identification process by the frequency. In this way, low-frequency components have less of an effect on the identification process.

The first solution was easy to apply. The rows and columns of the Sim-2 matrices corresponding to rigid-body modes were deleted, leaving a slightly smaller model with 21 symmetric and 21 antisymmetric modes.

To better explain the frequency weighting solution, recall from Dimitriadis and Cooper<sup>1</sup> that the Nissim and Gilyard system identification process is described by

$$\begin{pmatrix} q_{1}(\omega_{1}) & \dots & q_{m}(\omega_{1}) & j\omega_{1}q_{1}(\omega_{1}) & \dots & j\omega_{1}q_{m}(\omega_{1}) & -g(\omega_{1}) \\ q_{1}(\omega_{2}) & \dots & q_{m}(\omega_{2}) & j\omega_{2}q_{1}(\omega_{2}) & \dots & j\omega_{2}q_{m}(\omega_{2}) & -g(\omega_{2}) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ q_{1}(\omega_{nf}) & \dots & q_{m}(\omega_{nf}) & j\omega_{nf}q_{1}(\omega_{nf}) & \dots & j\omega_{nf}q_{m}(\omega_{nf}) & -g(\omega_{nf}) \end{pmatrix} \begin{pmatrix} \mathbf{K}^{T} \\ \mathbf{C}^{T} \\ \mathbf{F}^{T} \end{pmatrix} = \begin{pmatrix} \omega_{1}^{2}q_{1}(\omega_{1}) & \dots & \omega_{1}^{2}q_{m}(\omega_{1}) \\ \omega_{2}^{2}q_{1}(\omega_{2}) & \dots & \omega_{2}^{2}q_{m}(\omega_{2}) \\ \vdots & \vdots & \vdots & \vdots \\ \omega_{nf}^{2}q_{1}(\omega_{nf}) & \dots & \omega_{nf}^{2}q_{m}(\omega_{nf}) \end{pmatrix}$$
(1)

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where  $q_i(\omega_i)$  is the amplitude of the response of the *i*th mode at the jth frequency; K, C, and F are the stiffness, damping, and forcing matrices, respectively, multiplied by the inverse of the mass matrix; nf is the total number of frequencies used in the curve fit; and m is the total number of modes. The frequency weighting is achieved by multiplying each line of Eqs. (1) by the corresponding frequency  $\omega_i$ , that is,

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Velocity range, % of flutter speed	Damping fit, %	Flutter margin, %	Envelope, %	ARMAX, %	NGM, %
23–43	28.4		5.5	53.3	36.8
23–57	17.0	18.8	35.1	32.9	36.5
23-70	9.2	15.5	16.2	6.5	3.0
23-84	4.6	3.8	2.5	8.7	0.5
23-98	0.3	0.8	0.7	1.0	0.2
45-66	2.8	20.8	9.4	12.1	14.3
45-79	2.3	5.5	6.0	8.7	0.9
45-93	0.5		1.3	2.2	0.6
68–88	2.9		0.5	2.0	1.8

Table 1 Modulus of errors in flutter estimates, Sim-2 model

Table 2 Flutter predictions for the Sim-2 model, simulated flutter test

Method	Mean flutter speed estimate, kn	Mean error, %	
Damping fit	$394.15 \pm 5.88$	-1.0	
Envelope	$386.82 \pm 2.47$	-2.8	
Flutter margin	$432.82 \pm 5.38$	8.8	
ARMAX	$164.08 \pm 9.87$	-58.8	
NGM	$338.55 \pm 51.91$	-14.9	

error in the flutter predictions obtained by all of the methods for tests carried out at different frequency ranges. All of the flutter predictions were obtained from data contaminated with 5% rms simulated noise. Each calculation was repeated for 30 different sets of simulated noise sequences and the mean errors were calculated. Table 1 shows that the flutter predictions obtained using the NGM demonstrate reasonably high errors when the flight testing is performed far away from the critical condition. However, when at least one of the test speeds is within 30% of the critical speed, the method delivers very

$$\begin{pmatrix} \omega_{1}q_{1}(\omega_{1}) & \dots & \omega_{1}q_{m}(\omega_{1}) & j\omega_{1}^{2}q_{1}(\omega_{1}) & \dots & j\omega_{1}^{2}q_{m}(\omega_{1}) & -\omega_{1}g(\omega_{1}) \\ \omega_{2}q_{1}(\omega_{2}) & \dots & \omega_{2}q_{m}(\omega_{2}) & j\omega_{2}^{2}q_{1}(\omega_{2}) & \dots & j\omega_{2}^{2}q_{m}(\omega_{2}) & -\omega_{2}g(\omega_{2}) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \omega_{nf}q_{1}(\omega_{nf}) & \dots & \omega_{nf}q_{m}(\omega_{nf}) & j\omega_{nf}^{2}q_{1}(\omega_{nf}) & \dots & j\omega_{nf}^{2}q_{m}(\omega_{nf}) & -\omega_{nf}g(\omega_{nf}) \end{pmatrix} \begin{pmatrix} \mathbf{K}^{T} \\ \mathbf{C}^{T} \\ \mathbf{F}^{T} \end{pmatrix} = \begin{pmatrix} \omega_{1}^{3}q_{1}(\omega_{1}) & \dots & \omega_{1}^{3}q_{m}(\omega_{1}) \\ \omega_{2}^{3}q_{1}(\omega_{2}) & \dots & \omega_{2}^{3}q_{m}(\omega_{2}) \\ \vdots & \vdots & \vdots & \vdots \\ \omega_{nf}^{3}q_{1}(\omega_{nf}) & \dots & \omega_{nf}^{3}q_{m}(\omega_{nf}) \end{pmatrix} (2)$$

Note that, after weighting, Eqs. (2) can be expressed uniquely in terms of the modal velocities and accelerations  $\dot{q}_i$  and  $\ddot{q}_i$  because  $\omega_j q_i = -j\dot{q}_i$ ,  $j\omega_i^2 q_i = j\ddot{q}_i$ , and  $\omega_i^3 q_i = -\omega_j \ddot{q}_i$ .

#### Additional Improvement to the Implementation of NGM

Nissim and Gilyard<sup>2</sup> state that the NGM performs better when multiple forcing vectors are applied to the aircraft during the flight tests. This was interpreted by Dimitriadis and Cooper as simultaneous forcing vectors applied at different locations of the aircraft, for example, ailerons, rudder, and elevator. However, the original methodology in fact concerns excitation vectors applied consecutively at different locations. For example, for three forcing vectors, the procedure would be to carry out three consecutive tests, one with excitation through the aileron, one through the rudder, and one through the elevator. Then the system matrices are obtained from a single curve fit containing all three sets of measured responses and applied excitations. This latter approach is less sensitive to noise because it performs an indirect averaging of the response data, smoothing out some of the noise. The results presented in this paper were obtained through the application of the consecutive excitation technique. The excitation positions chosen were the aileron, elevator, and engine nacelles.

Finally, the excitation signals used by Dimitriadis and Cooper<sup>1</sup> were sine sweeps. The results presented in the present work were obtained from random excitation with limited frequency content. Such signals allow better identification of the Sim-2 system matrices.

#### **Results**

The results from the application of the NGM to the Sim-2 model are presented here together with the results from the other methods, in exactly the same format. Results for the Hancock model are not presented because these were already very good in the original publication.

First, the flutter predictions from the "Method Validation" section of Dimitriadis and Cooper<sup>1</sup> are presented in Table 1. These show the

high-quality flutter predictions, better than those obtained by any of the other methods.

Table 2 shows the flutter estimates obtained from simulated flight flutter tests of the Sim-2 model using all of the methods. The responses were again contaminated by 5% rms simulated noise. All simulated flutter tests were repeated with 30 different sets of noise sequences, and the mean flutter speed predictions and 5% confidence intervals were calculated for each method. It can be seen that the results obtained by the NGM show the highest variability, despite the fact that the mean error is much better than that obtained from the ARMAX-based method. The reason for this high variability is the low accuracy of the method at low subcritical airspeeds, as demonstrated in Table 1, which can cause the flight test to end prematurely.

#### Conclusions

The Conclusions section of the original paper<sup>1</sup> stated that the best flutter predictions for the Sim-2 model were obtained from the application of the damping fit method. This conclusion must be altered now that results from the NGM are available for this model. The damping fit method still delivers the best overall accuracy; however, at high subcritical airspeeds, the best flutter predictions are obtained from the Nissim and Gilyard technique. In a real flutter test, most of the test points are generally concentrated at the higher subcritical conditions, exactly the flight regime where NGM works best.

### References

<sup>1</sup>Dimitriadis, G., and Cooper, J. E., "Flutter Prediction from Flight Flutter Test Data," *Journal of Aircraft*, Vol. 38, No. 2, 2001, pp. 355–367.

<sup>2</sup>Nissim, E., and Gilyard, G. B., "Method for Experimental Determination of Flutter Speed by Parameter Identification," AIAA Paper 89-1324-CP, AIAA, Washington, DC, 1989.

<sup>3</sup>Nissim, E., and Gilyard, G. B., "Method for Experimental Determination of Flutter Speed by Parameter Identification," NASA TP-2923, June 1989.